

# Clinical applications of the areas under ESPVR

Rachad M. Shoucri

Department of Mathematics & Computer Science, Royal Military College of Canada, Kingston, ON K7K7B4, Canada

Corresponding to: Rachad M. Shoucri. Department of Mathematics & Computer Science, Royal Military College of Canada, Kingston, ON K7K7B4, Canada. Email: shoucri-r@rmc.ca.



Submitted May 23, 2013. Accepted for publication May 25, 2013.

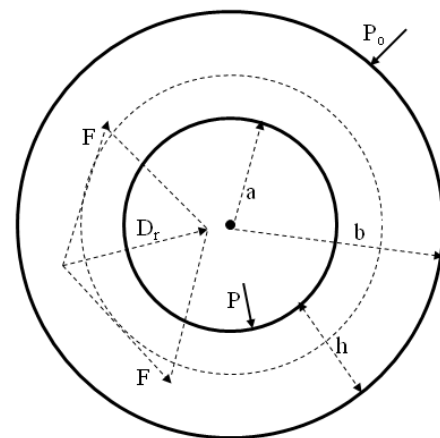
doi: 10.3978/j.issn.2223-3652.2013.05.05

Scan to your mobile device or view this article at: <http://www.thecdt.org/article/view/2042/3020>

## Indexes of cardiac performance

It has long been observed that the use of the ejection fraction (EF) to assess the condition of the heart offers serious limitations (1). It is estimated that half of the patients presenting with symptoms of cardiomyopathies and heart failure (HF) have preserved EF defined as  $EF > 50\%$ . It was shown in (2) that the areas under the end-systolic pressure-volume relation (ESPVR) in the heart ventricles are sensitive indexes to reflect the state of the myocardium, a review of some clinical applications of indexes derived from the ESPVR can be found in (3). An objective of the study by Doyle *et al.* (4) is precisely to show that areas under the ESPVR, or bivariate combination of areas with another index, can be used as a prognostically useful tool for studying cases of women with suspected myocardial ischemia.

The ESPVR is the relation between pressure and volume in the left or right ventricle when the myocardium reaches its maximum state of activation near end-systole (2,3). There have been several studies on the ESPVR (3,5-7), most of these studies have focused on the use of the maximum slope  $E_{max}$  and the volume axis intercept  $V_{om}$  of the ESPVR in order to assess the state of the myocardium. Because of the difficulty to calculate  $V_{om}$ , some researchers have tried to neglect  $V_{om}$  and to approximate  $E_{max} \approx$  (end-systolic pressure/end-systolic volume) (4). Such an approximation can only be justified if it is proven that the results obtained contain useful reliable clinical information. In what follows we introduce some relations that reflect the way the energetic of cardiac contraction is related to the areas under the ESPVR, and how the EF is also influenced by these areas. It may provide some background for the study published in (4).

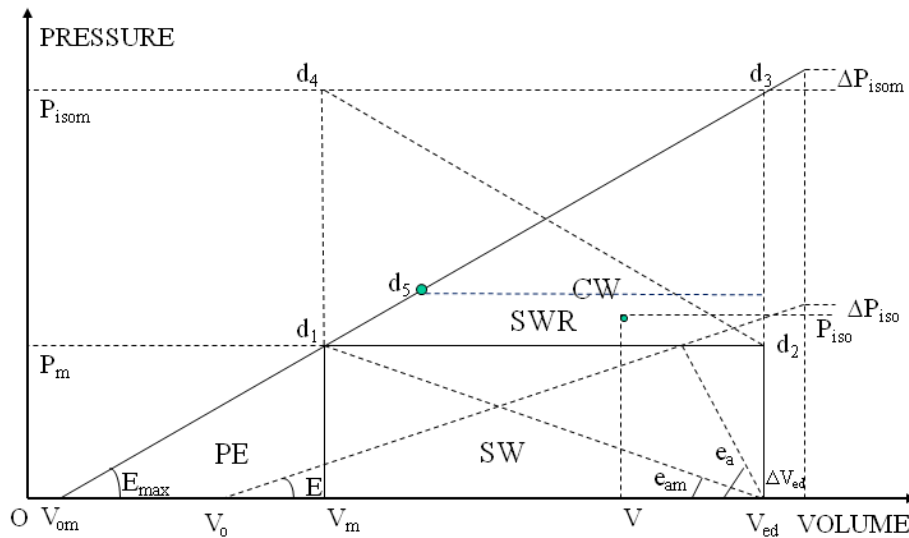


**Figure 1** Left ventricle represented by the cross-section of a thick-walled cylinder.  $D_r$ , active radial force/unit volume of the myocardium;  $P$ , left ventricular pressure;  $P_o$ , outer pressure (assumed zero) on the myocardium;  $a$ , inner radius;  $b$ , outer radius;  $h = b - a =$  thickness of the myocardium

## Mathematical formalism

### PVR

As in previous publications (8-11), the left ventricle is represented as a thick-walled cylinder contracting symmetrically (Figure 1). A radial active force  $D_r$  (force per unit volume of the myocardium) is developed by the myocardium during the contraction phase. The active pressure on the inner surface of the myocardium (endocardium) is given by  $\int_a^b D_r dr = P_{iso}$ , where  $a =$  inner radius of the myocardium,  $b =$  outer radius of the myocardium. We neglect inertia and viscous forces since they are relatively small. The equilibrium of forces on the



**Figure 2** Simplified PVR in the left ventricle,  $V_{ed}d_2d_1V_m$  represents the pressure-volume loop in a normal ejecting contraction (area SW). The ESPVR is represented by the line  $d_3V_{om}$  with midpoint  $d_5$  and slope  $E_{max}$ , the line with slope  $E$  is an intermediate position. The left ventricular pressure  $P_m$  is assumed constant during the ejection phase. The changes  $\Delta P_{iso}$  and  $\Delta P_{isom}$  correspond to  $\Delta V_{ed}$  according to the Frank-Starling mechanism

endocardium can then be expressed in the form:

$$P_{iso} - P = E (V_{ed} - V) \tag{1}$$

$P$  is the left ventricular pressure,  $V$  is the corresponding left ventricular volume,  $V_{ed}$  is the end-diastolic volume (the largest volume when  $dV/dt = 0$ ). The right-hand side of Equation [1] is the pressure on the endocardium resulting from the elastic deformation of the myocardium. When the elastance  $E$  reaches its maximum value  $E_{max}$  near end-systole (maximum state of activation of the myocardium), we can write Equation [1] as follows:

$$P_{isom} - P_m = E_{max} (V_{ed} - V_m) \tag{2}$$

We take  $V_m \approx V_{es}$  the end-systolic volume when  $dV/dt = 0$ .

**ESPVR**

Equations [1] and [2] are represented graphically in a simplified way in *Figure 2*. The ESPVR is a relation between  $P_m$  and  $V_m$  when the peak isovolumic pressure  $P_{isom}$  is kept constant, it is represented by the line  $d_3V_{om}$  with slope  $E_{max}$ .

During a normal ejecting contraction the PVR is represented by the rectangle  $V_{ed}d_2d_1V_m$ . Equation [2] can be split into the following form:

$$P_m = E_{max} (V_m - V_{om}) \tag{3}$$

$$P_{isom} = E_{max} (V_{ed} - V_{om}) \tag{4}$$

We can distinguish the following cases described by the

ratio  $E_{max}/e_{am}$  (maximum ventricular elastance/maximum arterial elastance) and the stroke volume  $SV \approx V_{ed} - V_m$  (see *Figure 2*).

(I) Normal physiological state of the heart, with  $d_1$  below  $d_3$  on the line  $d_3V_{om}$ . In this case we have  $SV > (V_{ed} - V_{om})/2$ , with  $E_{max}/e_{am} \approx 2$  and  $P_{isom}/P_m \approx 3$ . This case corresponds to maximum efficiency for  $O_2$  consumption by the myocardium.

(II) Mildly depressed state of the heart, with  $d_1$  and  $d_3$  coinciding. In this case we have  $SV \approx (V_{ed} - V_{om})/2$ , with  $E_{max}/e_{am} \approx 1$  and  $P_{isom}/P_m \approx 2$ . The stroke work  $SW$  reaches its maximum value  $SW_{max}$ .

(III) Severely depressed state of the heart, with  $d_1$  above  $d_3$  on the line  $d_3V_{om}$ . In this case we have  $SV < (V_{ed} - V_{om})/2$ , with  $E_{max}/e_{am} < 1$  and  $P_{isom}/P_m < 2$ .

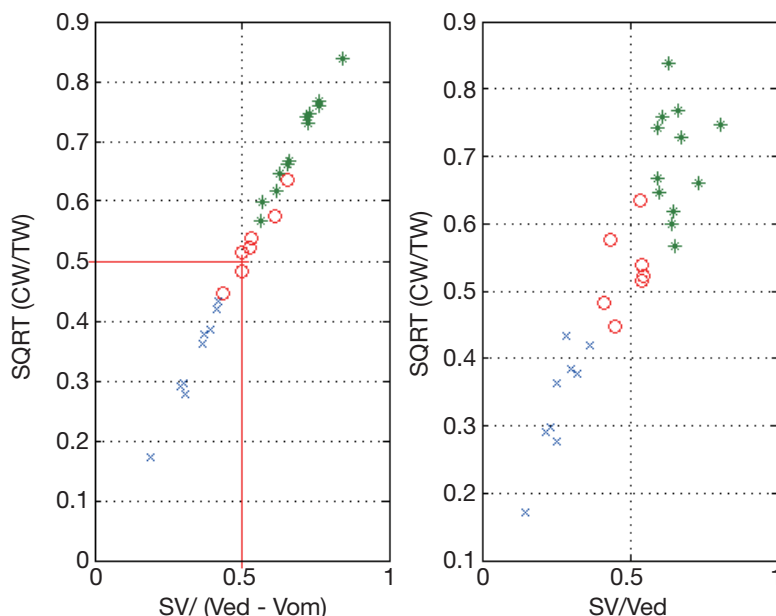
Notice from *Figure 2* that in cases (II) and (III) an increase in pressure  $P_m$  causes a decrease of the stroke work  $SW$ , resulting in cardiac insufficiency.

Experimental verification of these results can be found in (5) (left ventricle) and in (6) (right ventricle) for experiments on dogs, and in (7) for results obtained from patients.

We have the following areas under the ESPVR:

(I)  $SW = P_m (SV)$ , energy delivered to the systemic circulation;

(II)  $PE = P_m (V_m - V_{om})/2$ , potential energy apparently



**Figure 3** Graphical verification of [5] (left side), and similar relation with the  $EF = SV/V_{ed}$  (right side). Experimental data taken from Asanoi *et al.* (7). Data correspond to three clinical groups: (I)  $EF \geq 60\%$  '\*'; (II)  $40\% < EF < 59\%$  'o'; (III)  $EF < 40\%$  'x'

related to the internal metabolism of the myocardium;

(III)  $CW = (P_{isom} - P_m) SV/2$ , energy apparently absorbed by the passive medium of the myocardium;

(IV)  $SWR = SW_{max} - SW$ , stroke work reserve. It is the reserve energy that can be delivered to the systemic circulation when afterload represented by  $P_m$  is increased.

We have  $SW + PE + CW = TW$  the total area under ESPVR. One can derive the following relation for the stroke volume:

$$SV = (CW/TW)^{1/2} (V_{ed} - V_{om}) \quad [5]$$

Which shows how SV (and  $EF = SV/V_{ed}$ ) is determined by the areas under the ESPVR. When  $CW/TW = 1/4$  ( $d_1$  and  $d_5$  coincide in *Figure 2*), we get from Equation [5]  $SV = (V_{ed} - V_{om})/2$ . Experimental verification is shown in *Figure 3*. We also have:

$$E_{max}/e_{am} = 2 \times CW/SW \quad [6]$$

## Acknowledgements

*Disclosure:* The author declares no conflict of interest.

## References

- Sanderson JE. Heart failure with a normal ejection fraction. *Heart* 2007;93:155-8.
- Crottogini AJ, Willshaw P, Barra JG, et al. Inconsistency of the slope and the volume intercept of the end-systolic pressure-volume relationship as individual indexes of inotropic state in conscious dogs: presentation of an index combining both variables. *Circulation* 1987;76:1115-26.
- Burkhoff D, Mirsky I, Suga H. Assessment of systolic and diastolic ventricular properties via pressure-volume analysis: a guide for clinical, translational, and basic researchers. *Am J Physiol Heart Circ Physiol* 2005;289:H501-12.
- Doyle M, Weinberg N, Pohost GM, et al. Left ventricular energy model predicts adverse events in women with suspected myocardial ischemia: results from the NHLBI-sponsored women's ischemia syndrome (WISE) study. *Cardiovasc Diagn Ther* 2013;3:64-72.
- Burkhoff D, Sagawa K. Ventricular efficiency predicted by an analytical model. *Am J Physiol* 1986;250:R1021-7.
- Brimioulle S, Wauthy P, Ewalenko P, et al. Single-beat estimation of right ventricular end-systolic pressure-volume relationship. *Am J Physiol Heart Circ Physiol* 2003;284:H1625-30.
- Asanoi H, Sasayama S, Kameyama T. Ventriculoarterial coupling in normal and failing heart in humans. *Circ Res* 1989;65:483-93.
- Shoucri RM. Theoretical study of pressure-volume relation in left ventricle. *Am J Physiol* 1991;260:H282-91.
- Shoucri RM. Ventriculo-arterial coupling and the areas

- under the end-systolic pressure-volume relation. *Jpn Heart J* 1997;38:253-62.
10. Shoucri RM. Studying the mechanics of left ventricular

- contraction. *IEEE Eng Med Biol Mag* 1998;17:95-101.
11. Shoucri RM. ESPVR, ejection fraction and heart failure. *Cardiovasc Eng* 2010;10:207-12.

**Cite this article as:** Shoucri RM. Clinical applications of the areas under ESPVR. *Cardiovasc Diagn Ther* 2013;3(2):60-63. doi: 10.3978/j.issn.2223-3652.2013.05.05