Original Article

Improving multi-channel compressed sensing MRI with reweighted l_1 minimization

Ching-Hua Chang, Jim X. Ji

Department of Electrical and Computer Engineering, Texas A&M University, USA

Corresponding to: Jim X. Ji. Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX 77843-3128, USA. Email: jimji@tamu.edu.

Abstract: Integrating compressed sensing (CS) and parallel imaging (PI) with multi-channel receiver has proven to be an effective technology to speed up magnetic resonance imaging (MRI). In this paper, we propose a method that extends the reweighted l_1 minimization to the CS-MRI with multi-channel data. The method applies a reweighted l_1 minimization algorithm to reconstruct each channel image, and then generates the final image by a sum-of-squares method. Computer simulations based on synthetic data and *in vivo* MRI imaging data show that the new method can improve the reconstruction quality at a slightly increased computation cost.

Keywords: Compressed sensing MRI; reweighted l_1 minimization; multi-channel receive arrays; image reconstruction

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Introduction

Imaging speed in magnetic resonance imaging (MRI) is an important issue, especially in clinical settings because shortening the scan time can reduce the cost and increase throughput and patient's comfort. However, the data acquisition is practically limited by hardware capability and signal-to-noise ratio (SNR) factors. Compressed sensing (CS) is a method that allows a sparse signal to be reconstructed from a set of randomly under-sampled projection data (1,2). It has been demonstrated that CS is useful for speeding up MRI acquisition, where data is collected in the k-space, i.e., Fourier space (3).

Multi-channel imaging using array receiver system offers improved SNR (4,5) or accelerated speed with parallel imaging (PI). Therefore, integrating CS and PI are expected to further improve the MRI quality and/or speed (6-10). In doing so, CS and PI are coupled in a large linear system or decoupled in separated steps. In the latter case, CS algorithm is applied to each channel individually, then the final image can be reconstructed using the sensitivity encoding method (11) or a root-sum-of-squares method (12). In addition, the correlations of distributed compressed sensing has also been applied in the system to improve the image quality (13). In all the aforementioned methods, image reconstructions involves minimizing the l_1 norm of a sparse image representation in certain domains, such as the wavelet domain or total variation (TV). Since l_1 is an approximation of the sparsity measurements, i.e., l_0 norm of the sparse domain, there have been efforts to further improve l_1 minimization so that it will be closer to the l_0 minimization solution.

In this paper, we develop a method that reconstructs MRI image from multi-channel data in the CS framework with a reweighted l_1 minimization. The main feature of the new method is that it uses an iterative, reweighted l_1 minimization method to perform the CS reconstruction of multi-channel MRI data. The method was compared with two existing multi-channel CS reconstruction methods using computer simulations and *in vivo* MRI data. The results show that the proposed method can provide an improved reconstruction quality at a slightly increased computation cost. This paper is developed based on preliminary work presented in a conference abstract (14).

Methods

The array MR receiver system consists of a set of receiver

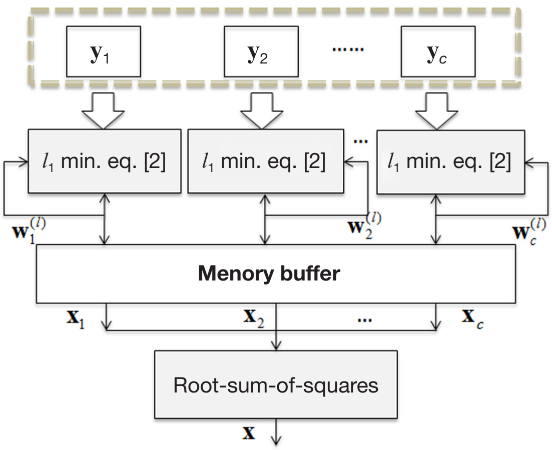

Figure 1 Reconstruction procedure for multi-channel receiver system using the l_1 reweighted minimization.

channels, which are individually connected to decoupled coil elements. With an array receiver system, a k-space data set $\mathbf{y}_k$, k = 1, 2,…, c, will be acquired from each channel. In applying CS reconstruction, each channel can be formulated as an underdetermined system, $\mathbf{y}_k=\mathbf{\Phi}\mathbf{x}_k$, where $\mathbf{\Phi}$ is an operator of randomly under-sampled Fourier Transform implemented by the phase-encoding and frequency-encoding gradients. The CS theory states that an image $\mathbf{x}_k$ can be recovered from the incomplete k-space data $\mathbf{y}_k$ if it is sufficiently sparse. However, even the image itself is not sparse, it can often be transformed to a sparser domain and there is high probability that the image can be recovered. A commonly used sparsifying transform is the gradient operators; i.e., the image reconstruction can be achieved by solving the following convex optimization problem,

$$\min_{\mathbf{x}_k} TV(\mathbf{x}_k) \text{ subject to } \mathbf{y}_k=\mathbf{\Phi}\mathbf{x}_k \quad k=1, 2,\ldots,c \qquad [1]$$

where $TV(\mathbf{x}_k) = \sum_{\substack{1\le i\le m\\1\le j\le n}} \left\|(D\mathbf{x}_k)_{i,j}\right\|_2$, where $(D\mathbf{x}_k)_{i,j}$ represents the forward difference between adjacent pixels defined as $(x_{i+1,j}-x_{i,j},\ x_{i,j+1}-x_{i,j})$. Here, total variation (TV) is considered as the l_1 norm of the magnitudes of the gradients. This formulation follows the method described in (12).

After all channels images are reconstructed. They are combined using a root-sum-of-squares method. The overall reconstruction procedure is shown in *Figure 1*. As shown, under-sampled k-space data is fed to the use of l_1 minimization algorithm, whose outputs are recursively calculated as the weights of the next iteration and finally produce the final image.

In this paper, we utilize the reweighted l_1 minimization algorithm (15) to enhance the CS image reconstruction from multi-channel data. To solve the minimization problem in Eq. [1], it is rewritten as a second-order cone problem with weights:

$$\min_{t_k,\mathbf{x}_k} \sum_{\substack{1\le i\le m\\1\le j\le n}} t_{k_{i,j}} \quad \text{subject to} \quad w_{k_{ij}} \left\|(D\mathbf{x}_k)_{i,j}\right\|_2 \le t_{k_{i,j}}$$

$$\mathbf{y}_k=\mathbf{\Phi}\mathbf{x}_k$$

where the weights are set to be inversely proportional to the signal magnitude. Based on the theory of reweighted l_1 minimization, the larger entries of $\mathbf{w}_k$, i.e., where signal magnitude is close to zero, will discourage small entries of the reconstructed image $\mathbf{x}_k$. In the proposed method, small weights are calculated from the previous reconstructed images. As a result, the weights can be considered as iterative parameters in the convex relaxation to improve the image reconstruction.

Specifically, each image $\mathbf{x}_k$ is reconstructed as follows.

(I) Set the iteration count, l=1 and the initial weight, $w_{i,j}^{(1)} = 1$ for i=1,…,m and j=1,…,n. Note that $w_{i,j}^{(1)}$ is the weight on pixel (i, j).

(II)Solve the weighted l_1 minimization problem

$$\mathbf{x}_k^{(l)} = \arg\min \sum_{\substack{1\le i\le m\\1\le j\le n}} w_{k_{i,j}}^{(l)} \left\|(D\mathbf{x}_k)_{i,j}\right\|_2 \text{ subject to } \mathbf{y}_k=\mathbf{\Phi}\mathbf{x}_k \qquad [2]$$

This was performed using a home-made Matlab program by modifying the l_1-magic software package (16).

(III) Update the weights:

$$w_{i,j}^{(l+1)} = 1/\left(\left\|(D\mathbf{x}_k^{(l)})_{i,j}\right\|_2 + \epsilon\right) \qquad [3]$$

The parameter ε is a small positive number to prevent zero-valued denominator. In this paper, it is set to 0.2 of the normalized received data.

(IV) If $l<l_{max}$, increase l and go to step 2.

Finally, all the reconstruction images are combined by the root-sum-of-squares of all channel images.

$$\mathbf{x}(i,j) = \sqrt{\sum_{k=1}^{c} |\mathbf{x}_k^{(l_{max})}(i,j)|^2} \qquad [4]$$

To test the proposed method, both simulated and *in vivo* data were used. The k-space data of four channels were simulated using the 'Shepp-Logan' phantom with an image size of 128×128. The individual channel sensitivities are assumed to be shifted 2-dimension Gaussian functions. The individual channel data were under-sampled in the k-space with radial sampling pattern. The under-sampling factor was about 15%, which meant only 15% of the total data were used in reconstructions. Finally, an 8-channel *in vivo* brain

Table 1 NMSE of the image reconstruction in the simulated 4-channel phantom study

NMSE	*Ch1*	*Ch2*	*Ch3*	*Ch4*
TV (l_1 minimization)	0.015	0.041	0.036	0.016
Proposed (with reweighted l_1 minimization)	0.011	0.026	0.025	0.014

NMSE, normalized means square error; TV, total variation.

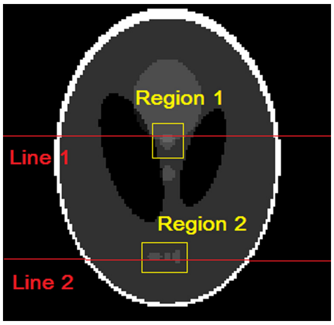

Figure 2 Original phantom image with selected regions and lines for comparisons.

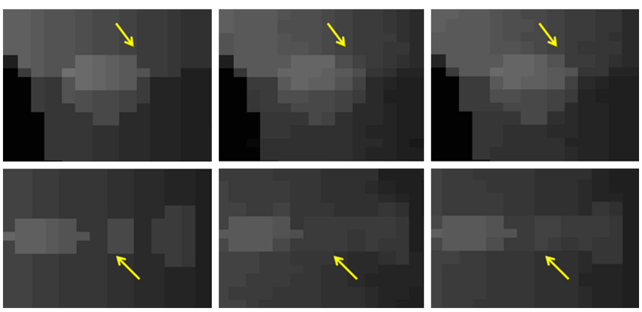

Figure 3 Reconstruction details of the Zoom-in region 1 (top row images) and region 2 (bottom row images) in the channel-two image: (left) reference from the fully sampled data (middle) with conventional l_1 minimization (TV) (right) with the proposed reweighted l_1 minimization.

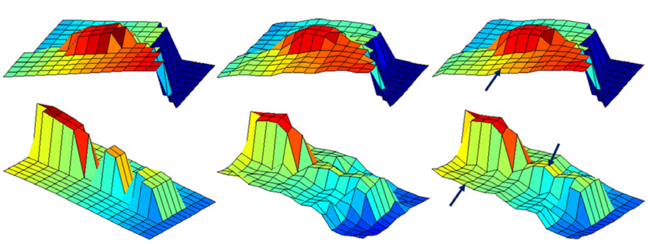

Figure 4 Surface plots of the corresponding zoom-in regions shown in *Figure 3*: (left) reference from the fully sampled data (middle) with conventional l_1 minimization (TV) (right) with the proposed reweighted l_1 minimization.

MR data set was acquired and tested. The k-space data with an image size of 256×256 from each-channel were acquired in full field-of-view, i.e., without under-sampling. Then, the radial sampling was simulated by decimation with an under-sampling factor of 25%. Based on the same sampling factor, the reconstruction image using the proposed method was compared with two methods: (I) conventional TV minimization (l_1 minimization with no reweighted iterations); and (II) a method combined in (9), which combine CS with SPACE-RIP (Sensitivity Profiles from an Array of Coils for Encoding and Reconstruction In Parallel).

The normalized means square error (NMSE) was used to evaluate the performance and defined as follows

$$\mathrm{NMSE} = \left\|\hat{\mathbf{x}}_k - \mathbf{x}_k^{(l_{max})}\right\|_2 / \|\hat{\mathbf{x}}_k\|_2 \qquad [5]$$

Note that $\hat{\mathbf{x}}_k$ is the referenced image, which is reconstructed from the fully sampled data in the k-th channel.

Results

To show the quantitative improvement of the proposed approach the NMSE of the reconstructions by the conventional TV (l_1 minimization) and the proposed method is shown in *Table 1*. It shows that the proposed method has a lower NMSE than the conventional l_1 minimization algorithm since low NMSE represents less reconstruction error; the proposed method is superior in this study. *Figures 2-5* show the images and reconstruction details in the simulated phantom study. *Figure 2* indicates two regions and two lines on the original phantom study, which are used to compare the reconstructed details and resolutions. The comparisons of the reconstruction details are show in *Figures 3* and *4*. As the highlight region 1 and region 2 shown in *Figure 3*, the proposed method can recover more details of the edges pointed by the arrows. This is also illustrated in *Figure 4*, which displays the surface plots of the same corresponding zoom-in images shown in *Figure 3*. The three-dimension angle of view is also indicated along the arrows shown in *Figure 3*. Besides recovering sharper edges, it is observed that the proposed method can eliminate the staircase artifacts around smooth area noted by these arrows of *Figure 4*. In addition, the difference between the original image and the reconstructed image, i.e., reconstruction errors along line 1 and line 2, are shown in *Figure 5*. Again, it demonstrates the proposed method yields the reduced reconstruction error.

Figure 6 compares the image reconstructions in an 8-channel *in vivo* brain imaging experiment. Here, the left

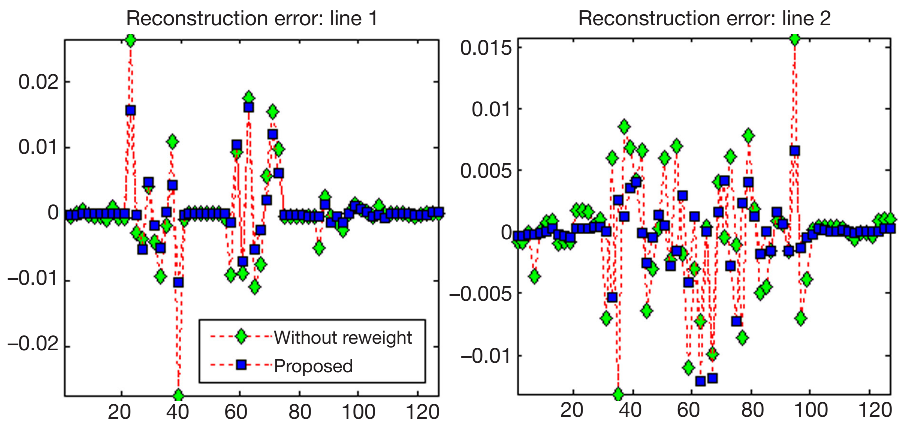

Figure 5 Reconstruction errors (differences between the original image and reconstructed image) along (left) line 1 (right) line 2.

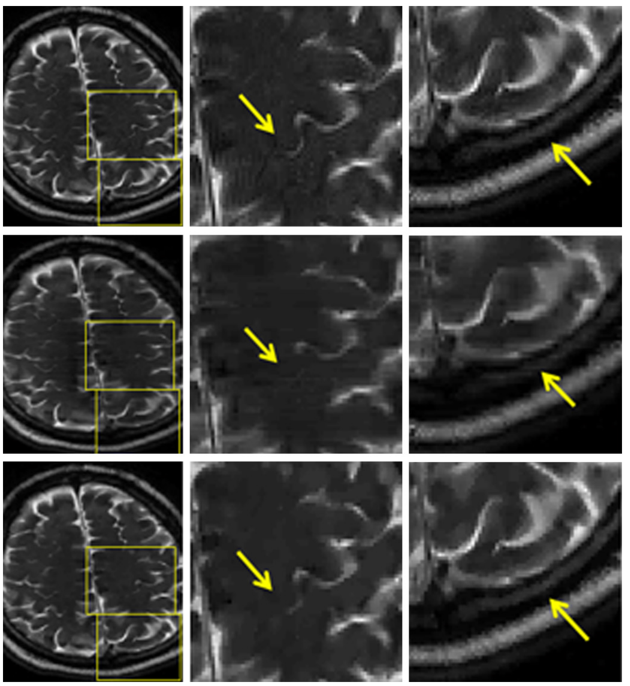

Figure 6 Images reconstructed from the 8-channel *in vivo* data using (TOP) sum-of-squares from fully sampled data, (middle) the method in (9), and (bottom) the proposed method.

column represents the reconstructed images from the fully sampled data, the method in (9) , where CS is integrated into a large linear system of multiple receiver coils, and the proposed method, respectively. The middle and right columns show the zoom-in views of the regions highlighted. To facilitate visualization, arrows are placed at the area where significant differences can be observed. As can be seen, higher fidelity in details and sharper features are obtained with the proposed method. Note that all images in the middle and bottom rows are reconstructed from 25% of the fully sampled data.

A comparison between the proposed method and the conventional TV minimization is shown in *Table 2* (only even channels are shown). The performance in terms of NMSE is shown. One can see that the proposed method has smaller quantitative reconstruction error.

Table 2 NMSE of the image reconstruction in the 8-channel *in vivo* imaging experiment

NMSE	*Ch2*	*Ch4*	*Ch6*	*Ch8*
TV (l_1 minimization)	0.092	0.086	0.096	0.086
Proposed (with reweighted l_1 minimization)	0.091	0.086	0.093	0.083

NMSE, normalized means square error; TV, total variation.

Discussion

A new improved reconstruction method for compressive sensing MRI with multi-channel phased array data was presented. In this method, the image is reconstructed using the reweighted l_1 minimization algorithm in a channel-by-channel fashion. The simulated experimental results show that the new method can provide an improved image quality from the same data. On the other hand, the new algorithm requires more iterations than the conventional l_1 minimization algorithm. This might pose a problem when immediate delivery of images is preferred. In such cases, using multi-core processors such as graphic processing unit (GPU) can be applied to parallelize the reconstruction and to shorten the reconstruction time. The proposed method can also be applied to the other CS methods where l_1 minimization is used.

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