Supplementary

Appendix A

When tissue is under finite deformations, ultrasound wave velocities c depends on the applied stress α. This relationship is described by the acoustoelastic theory and involves second-order elastic constants (Lamé parameters λ and μ) and third-order elastic constants (Murnaghan constants l, m, and n). The latter describe the nonlinear elastic properties of tissue. This appendix introduces the equations of wave velocities in tissue under uniaxial compressive stress.

For simplicity, we consider an isotropic and lossless medium. The speed of sound of longitudinal waves traveling parallel to the applied stress is given by

$$\rho c_{SoS}^2 = \lambda + 2\mu - \frac{\sigma}{3\lambda + 2\mu}\left[2l + \lambda + \frac{\lambda + \mu}{\mu}\left(4m + 4\lambda + 10\mu\right)\right] \qquad [1]$$

where ρ is the density of unstressed tissue (27). Similarly, the velocity of shear waves traveling perpendicular to the stress, with polarization parallel to it, is

$$\rho c_{SWS}^2 = \mu - \frac{\sigma}{3\lambda + 2\mu}\left[m + \frac{\lambda n}{4\mu} + \lambda + 2\mu\right] \qquad [2]$$

Due to the anisotropic nature of the applied stress, an unstressed isotropic medium will exhibit direction-dependent wave speeds under finite deformations. We refer the reader to (27) for a complete description of acoustoelastic equations. These equations show that squared velocities are linearly related to the applied stress. In general, we can express this relationship as

$$c^2 = c_0^2 + \sigma A\left(\lambda, \mu, m, n, l\right) \quad [3]$$

where c denotes the stress-dependent velocity of either longitudinal or shear waves, c_0 is the corresponding velocity for undeformed tissue, and A is the acoustoelastic parameter, namely the slope of the linear relationship containing the third-order elastic constants. The estimation of the acoustoelastic parameter provides access to tissue elastic nonlinear properties.

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