Appendix 1

Derivation of Eq. [2]

The probability density function (PDF) $p_{|\tilde{A}|}\left(|\tilde{A}|\right)$ of the amplitude component of noiseless OCT signal $|\tilde{A}|$ follows the Rayleigh distribution (14):

$$p_{|\tilde{A}|}\left(|\tilde{A}|\right) = \frac{2|\tilde{A}|}{v^2} exp\left(-\frac{|\tilde{A}|^2}{v^2}\right), \quad \text{[S1]}$$

According to the definition, $a_0 = \frac{|\tilde{A}|}{s}$ and $iSNR = \frac{s^2}{s^2+v^2}$, the PDF of a_0 can be expressed in terms of a_0 and $iSNR$:

$$p_{A_0}\left(a_0\right) = \frac{2iSNR \cdot a_0}{1-iSNR} e^{-\frac{iSNR}{1-iSNR}a_0^2}, \quad \text{[S2]}$$

Derivation of Eq. [3]

For signal in static region, in the temporal dimension, the resultant OCT signal $\tilde{X}$ is the sum of the constant phasor $\tilde{A}$ and the white Gaussian noise $\tilde{n}$. We assume the direction of the constant phasor as the real-axis for simplicity. The joint PDF $p_{\tilde{X}}\left(x,y\right)$ for the real and imaginary part of $\tilde{X}$ satisfies (14,34):

$$p_{\tilde{X}}\left(x,y\right) = \frac{1}{\pi s^2} exp\left[-\frac{\left(x-|\tilde{A}|\right)^2 + y^2}{s^2}\right], \quad \text{[S3]}$$

where x and y are defined as the real and imaginary part of $\tilde{X}$. By transforming Eq. [S3) into the polar coordinate, the PDF $p_{\tilde{X}}\left(|\tilde{X}|,\theta\right)$ of $\tilde{X}$ can be expressed in terms of the amplitude $|\tilde{X}|$ and angle θ of $\tilde{X}$:

$$p_{\tilde{X}}\left(|\tilde{X}|,\theta\right) = \frac{1}{\pi s^2} exp\left[-\frac{|\tilde{X}|^2 + |\tilde{A}|^2 - 2|\tilde{X}||\tilde{A}|\cos\theta}{s^2}\right]. \quad \text{[S4]}$$

The marginal PDF $p_{|\tilde{X}|}\left(|\tilde{X}|\right)$ of the amplitude $|\tilde{X}|$ can be obtained by integrating over θ:

$$\begin{aligned} p_{|\tilde{X}|}\left(|\tilde{X}|\right) &= \int_{-\pi}^{\pi} p_{\tilde{X}}\left(|\tilde{X}|,\theta\right) d\theta \\ &= \frac{1}{\pi s^2} exp\left(-\frac{|\tilde{X}|^2 + |\tilde{A}|^2}{s^2}\right) \int_{-\pi}^{\pi} exp\left(\frac{2|\tilde{X}||\tilde{A}|\cos\theta}{s^2}\right) d\theta = \frac{2}{s^2} exp\left(-\frac{|\tilde{X}|^2 + |\tilde{A}|^2}{s^2}\right) I_0\left(\frac{2|\tilde{X}||\tilde{A}|}{s^2}\right) \end{aligned} \quad \text{[S5]}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. According to the definition, $x_0 = \frac{|\tilde{X}|}{s}$ and $a_0 = \frac{|\tilde{A}|}{s}$, Eq. [S5) can be simplified as:

$$p_{X_0|A_0=a_0}\left(x_0\right) = 2x_0 e^{-x_0^2-a_0^2} I_0\left(2x_0 a_0\right). \quad \text{[S6]}$$

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Derivation of Eq. [7]

By setting $iSNR \to 1$ in Eq. [6], the asymptotic decorrelation $\overline{D_{dy}}$ can be calculated as following:

$$\overline{D_{dy}} \to 1-\frac{\pi}{4}\int_0^{+\infty}\left[L_{0.5}\left(-a_0^2\right)\right]^2\delta\left(a_0\right)da_0 = 1-\frac{\pi}{4}\left[L_{0.5}\left(-a_0^2\right)\right]^2 = 1-\frac{\pi}{4}\approx 0.22, a.s., \qquad [S7]$$

where the Rayleigh distribution $p_{A_0}\left(a_0\right)=\frac{2iSNR\cdot a_0}{1-iSNR}e^{-\frac{iSNR}{1-iSNR}a_0^2}$ approaches the delta function $\delta(a_0)$ when iSNR approaches 1.

Derivation of Eq. [10]

Given the normalized amplitude of the noiseless OCT signal a_0, the PDF of the decorrelation $D\left(x_0\left(t\right)\right)=1-iSNR^* x_0\left(t\right)\sqrt{2SNR^*-x_0^2\left(t\right)}$ for OCT signals in static region can be expressed as a function of $x_0(t)$:

$$p_{D\left(x_0(t)\right)|SNR^*}\left(x_0\left(t\right)\right)=p_{X_0|A_0=a_0}\left(x_0\left(t\right)\right)p_{X_0|A_0=a_0}\left(x_0\left(t+1\right)\right)\sqrt{1+\left(\frac{\partial x_0\left(t+1\right)}{\partial x_0\left(t\right)}\right)^2}. \qquad [S8]$$

Furthermore, a_0 follows a Rayleigh distribution determined by the local SNR. Taking this factor into consideration, the complete expression of Eq. [S8] is given by:

$$p_{D\left(x_0(t)\right)|SNR^*}\left(x_0\left(t\right)\right)=\int_0^1 p_{isnr}\left(iSNR\right)diSNR\int_0^{+\infty}p_{A_0}\left(a_0\right)da_0$$

$$p_{X_0|A_0=a_0}\left(x_0\left(t\right)\right)p_{X_0|A_0=a_0}\left(x_0\left(t+1\right)\right)\sqrt{1+\left(\frac{\partial x_0\left(t+1\right)}{\partial x_0\left(t\right)}\right)^2}. \qquad [S9]$$

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