Supplementary

Appendix 1

Algorithm 1 Higher-order singular value decomposition (HOSVD) for Pt-LRT reconstruction

INPUT: fourth-order tensor $\Gamma \in C^{N_1 \times N_2 \times N_3 \times N_4}$ with dimensions (N_1,N_2,N_3,N_4) and the regularization parameter λ=[λ_1, λ_2, λ_3, λ_4]

ALGORITHM:

Unfold the tensor along its single modes:

$T_{(1)}$: reshapes Γ into an $N_1 \times (N_2 \times N_3 \times N_4)$complex matrix.

$T_{(2)}$: reshapes Γ into an $N_2 \times (N_1 \times N_3 \times N_4)$ complex matrix.

$T_{(3)}$: reshapes Γ into an $N_3 \times (N_1 \times N_2 \times N_4)$ complex matrix.

$T_{(4)}$: reshapes Γ into an $N_4 \times (N_1 \times N_2 \times N_3)$ complex matrix.

(2) Compute the complex SVD of $T_{(n)}$ (n = 1, 2, 3, 4) and obtain the orthogonal matrices $U_{(1)}$, $U_{(2)}$, $U_{(3)}$ and $U_{(4)}$ from the n-mode signal subspace,

(3) Compute the complex core tensor $\mathcal{G}$ related by

$$\mathcal{G}=\Gamma \times_1 \mathbf{U}_{(1)}^{\boldsymbol{H}} \times_2 \mathbf{U}_{(2)}^{\boldsymbol{H}} \times_3 \mathbf{U}_{(3)}^{\boldsymbol{H}} \times_4 \mathbf{U}_{(4)}^{\boldsymbol{H}}$$

which is equivalent to its unfolding forms:

$$\boldsymbol{G}_{(\boldsymbol{n})} = \mathbf{U}_{(\boldsymbol{n})}^{\boldsymbol{H}} \mathbf{T}_{(\boldsymbol{n})} \left[\mathbf{U}_{(i)} \otimes \mathbf{U}_{(j)} \right]$$, with 1 ≤ n ≤ 4 and € ≠ j ≠ n

where ⊗ represents the Kronecker product.

(4) Compute the high-order singular value truncation (soft thresholding on $G_{(n)}$:

$$ST(p)_{\boldsymbol{G}_{(n)}} = \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \max\left(0, |p| - \boldsymbol{\lambda}_{\boldsymbol{n}}\right)$$

where *p* is an element of the $G_{(n)}$.

(5) Construct back the filtered tensor with the n-mode (n = 1, 2, 3, 4) unfolding matrix , calculated as follows:

$$\mathbf{T}_{(\boldsymbol{n})}^{denoise} = \mathbf{U}_{(\boldsymbol{n})} \mathcal{G} [\boldsymbol{U}_{(i)} \otimes \boldsymbol{U}_{(j)}]^H$$ with 1 ≤ n ≤ 4 and i ≠ j ≠ n

OUTPUT: The denoised tensor is obtained by folding.

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