Supplementary

Appendix 1

Overview of comparative algorithms

(I) The filtered-back projection (FBP) method:

The projection data obtained from the simulation is enhanced by applying a filter (Ram-Lak filter) to emphasize the high-frequency information in the projection data. The filtered projection data is then back-projected, and the back-projection results from all angles are superimposed to form the final reconstructed image.

(II) The TV-based method (54):

Reconstruction model:

$$\min_{x_s} \frac{1}{2}\sum_{s=1}^{S}\left\|Ax_s - p_s\right\|_2^2 + \lambda\sum_{s=1}^{S}\left\|\nabla x_s\right\|_1 .$$

ADMM-based algorithm:

Introducing an auxiliary variable $u_s = x_s$, and construct the corresponding Lagrangian function:

$$\mathcal{L}_A(x_s, u_s; \Lambda) = \frac{1}{2}\sum_{s=1}^{S}\left\|Ax_s - p_s\right\|_2^2 + \lambda\sum_{s=1}^{S}\left\|\nabla u_s\right\|_1 + \beta\sum_{s=1}^{S}\left\|x_s - u_s + \frac{\Lambda}{\beta}\right\|_2^2 .$$

Then decompose $\mathcal{L}_A(x_s, u_s; \Lambda)$ into two subproblem x_s and u_s, solve them iteratively.

(III) The reconstruction method named SBM_L0 (*Figure S1*) proposed in article (27):

Reconstruction model:

$$\min_{x_s, Z, E} \frac{1}{2}\sum_{s=1}^{S}\left\|Ax_s - b_s\right\|_2^2 + \lambda\sum_{s=1}^{S}\left\|\nabla x_s\right\|_0 + \beta R(Z) + \frac{\rho}{2}\left\|X - EZ\right\|_F^2 ,$$
$$s.t. \quad E^T E = I_k$$

Where X is the group of all-channel computed tomography (CT) images, The Frobenius norm term represents the difference between subspace decomposition EZ and multi-energy computed tomography (MECT) images X. $R(Z)$ represents the regularization term (BM3D) on eigen images tensor, λ, β, and ρ are the nonnegative parameters to balance the data fidelity and regularization term.

 https://dx.doi.org/10.21037/qims-24-1248

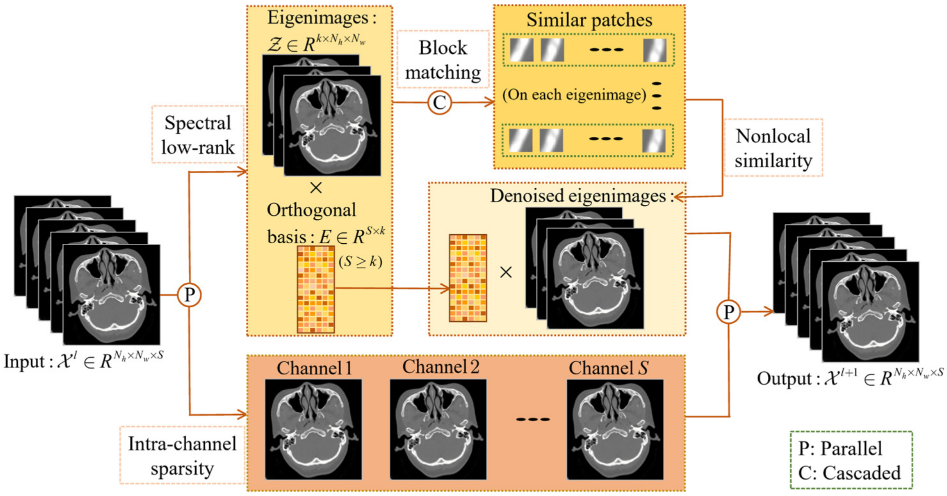

Figure S1 The flowchart of the SBM_L0 method. SBM_L0, subspace decomposition coming block-matching method.

ADMM-based algorithm:

$$\arg\min_{\mathcal{X}} \frac{1}{2}\sum_{s=1}^{S}\left\|Ax_s - b_s\right\|_2^2 + \frac{\rho}{2}\left\|X - E^{l-1}Z^{l-1}\right\|_F^2 + \frac{\eta}{2}\sum_{s=1}^{S}\left\|x_s - u_s^j - v_s^j\right\|_2^2,$$

$$\arg\min_{\mathcal{U}} \lambda\sum_{s=1}^{S}\left\|\nabla u_s\right\|_0 + \frac{\eta}{2}\sum_{s=1}^{S}\left\|x_s^{j+1} - u_s - v_s^j\right\|_2^2,$$

$$\mathcal{V}^{j+1} = \mathcal{V}^j + \mathcal{U}^{j+1} - \mathcal{X}^{j+1}$$

(IV) The FTNN method (*Figure S2*) (22):

Reconstruction model:

$$\min_{\mathcal{X}} \mathcal{R}(\mathcal{T}(\mathcal{X})) \quad s.t. \sum_{s=1}^{S} \|Ax_s - y_s\|_2^2 \leq \varepsilon \, .$$

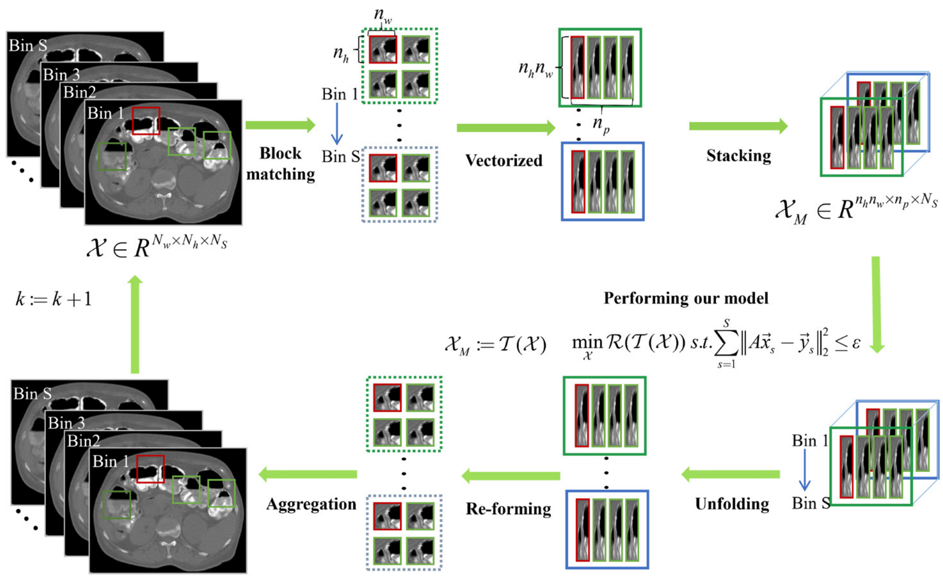

Figure S2 The flowchart of the FTNN method. FTNN, framelet tensor sparsity with block-matching method.

https://dx.doi.org/10.21037/qims-24-1248

As shown in the figure above, $\mathcal{X}_M = \mathcal{T}(\mathcal{X})$ is a tensor of non-local similar image patches extracted from the image tensor $\mathcal{X}$, a regularization of framelet tensor nuclear norm (FTNN) is applied to measure its low rankness, which can be expressed as

$$\mathcal{R}(\mathcal{X}_M) = \|\mathcal{X}_M\|_{FTNN} = \sum_{i=1}^{wN_S} \left\| [fold_3(W\mathcal{X}_{M(3)})]^{(i)} \right\|_* .$$

ADMM-based algorithm:

Introducing an auxiliary variable $\mathcal{X} = \mathcal{Z}$, the corresponding augmented Lagrangian function is

$$L(\mathcal{X}, \mathcal{Z}, \Lambda) = \mathcal{R}(\mathcal{T}(\mathcal{Z})) + \sum_{s=1}^{S} \delta_{\Omega_\varepsilon}(x_s) + \frac{\beta}{2} \left\| \mathcal{X} - \mathcal{Z} + \frac{\Lambda}{\beta} \right\|_F^2 .$$

Decompose $L(\mathcal{X}, \mathcal{Z}, \Lambda)$ into two subproblem $\mathcal{X}$ and $\mathcal{Z}$, solve them iteratively.

(V) The ITS_TV method (23):

Reconstruction model:

$$\lim_{\mathcal{X}}(\alpha \mathcal{R}(\mathcal{T}(\mathcal{X})) + \sum_{s=1}^{S} \|x_s\|_{TV}) \quad s.t. \sum_{s=1}^{S} \|Ax_s - y_s\|_2^2 \le \varepsilon \cdot$$

$\mathcal{X}_M = \mathcal{T}(\mathcal{X})$ is a tensor of non-local similar image patches extracted from the image tensor $\mathcal{X}$ like the *Figure S2* in the FTNN method. However, the model here uses a different tensor rank representation $\mathcal{R}(\mathcal{X}_M) = \|S\|_0 + t\prod_{i=1}^{I} rank(\mathcal{X}_M^{(i)})$.

ADMM-based algorithm:

Introducing an auxiliary variable $\mathcal{X} = \mathcal{Z}$, the corresponding augmented Lagrangian function is

$$L(\mathcal{X}, \mathcal{Z}, \Lambda) = \alpha \mathcal{R}(\mathcal{T}(\mathcal{Z})) + \sum_{s=1}^{S} \|x_s\|_{TV} + \sum_{s=1}^{S} \delta_{\Omega_\varepsilon}(x_s) + \frac{\beta}{2} \left\| \mathcal{X} - \mathcal{Z} + \frac{\Lambda}{\beta} \right\|_F^2 .$$

Decompose $L(\mathcal{X}, \mathcal{Z}, \Lambda)$ into two subproblem $\mathcal{X}$ and $\mathcal{Z}$, solve them iteratively.

https://dx.doi.org/10.21037/qims-24-1248