

Appendix 1 Detailed Derivation and Rationale of the Energetic Indices

1. Physiologic Foundation

Hydraulic work (W) represents the mechanical energy that the ventricle transfers to the arterial system during one cardiac cycle:

$$[W = P \times V]$$

where (P) is mean arterial pressure (Pa) and (V) is the volume of blood ejected per beat (m^3). In the cardiovascular system, this work maintains forward flow against vascular impedance. The Beat Work Index (BWI), Cardiac Power Index (CPI), and Arterial Elastance Index (AEI) were designed to quantify these components non-invasively using standard Doppler echocardiography and blood-pressure data.

2. Standardization of LVOT Geometry

Direct LVOT area measurement introduces variability because $area = \pi \times (D/2)^2$ magnifies any error in the LVOT diameter (D). Moreover, LVOT diameter increases with somatic growth, whereas LVOT VTI remains relatively constant across body sizes. To minimize geometric error and facilitate unit conversion, the LVOT was standardized to a fixed diameter of 2 cm, giving a

$$LVOT\ area = \pi \times \left(\frac{D}{2}\right)^2$$

This constant acts as a *geometric scaling factor* that:

- normalizes results across age and body size,
- eliminates the squared-error term in area measurement, and
- allows conversion of displacement \times pressure into physical energy units (J).

3. Derivation of the Constant ($k = 0.00041884$)

Hydraulic work per beat = Pressure \times Volume ($W = P \times V$).

To express the product of MAP ($mmHg$) \times VTI (cm) in joules, three conversions are required:

Step	Conversion	Factor
Pressure	1 mmHg = 133.322 N/m ² (Pa)	133.322
Area	Standard LVOT = 3.1416 \times 10 ⁻⁴	3.1416 \times 10 ⁻⁴
Volume	1 cm ³ = 1 \times 10 ⁻⁶	1 \times 10 ⁻⁶

$$k = 133.322 \times 3.1416 \times 10^{-6} = 4.1884$$

$$k = 4.1884 \times 10^{-4} = 0.00041884\ J / mmHg \times cm$$

4. Definition and Interpretation of the Energetic Indices

4.1 Beat Work Index (BWI) – energy per beat

$$BWI\ (J) = LVOT\ VTI\ (cm) \times MAP\ (mmHg) \times 0.00041884$$

Meaning: energy transferred from the ventricle to the arterial system per beat.

4.2 Cardiac Power Index (CPI) – energy per second

$$CPI\ (W) = \frac{BWI \times HR}{60\ sec}$$

Where HR=heart rate (beats per minute).

CPI integrates beat-level work over time, expressing the rate of energy transfer ($J \cdot s^{-1} = W$).

BWI captures *how strong each beat is*; CPI describes *how much energy per second* the heart delivers.

4.3. Arterial Elastance Index (AEI) – impedance or afterload

Arterial elastance represents afterload as the ratio of generated pressure to ejected volume:

$$AEI = \frac{MAP}{\text{Indexed Stroke Volume}}$$

With Indexed Stroke Volume = VTI \times 3.1416 cm² \times (1 mL / 1 cm³).

Thus, AEI reflects how many mmHg are required to eject each mL of blood, a practical non-invasive indicator of arterial stiffness and ventricular-arterial coupling.

5. Example Calculation

For representative postoperative child:

MAP=70 mmHg,

VTI = 20 cm,

HR=100 bpm.

$$BWI (J) = 20 \times 70 \times 0.00041884$$

$$BWI (J) = \frac{0.586 \text{ J/beat}}{0.586 \times 100}$$

$$CPI (W) = \frac{60 \text{ seg}}{70}$$

$$CPI (W) = 0.98 W$$

$$AEI = \frac{70}{20 \times 3.1416 \times 1}$$

$$AEI = 1.11 \text{ mmHg/mL}$$